

## TEST 2

(Math 140-A)

Solve the following problems. Show all your work in the space under each problem.

1. (a) **True or False:** The vector  $\vec{u} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$  is an eigenvector of  $A = \begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix}$ .

(b) Find the eigenvalues of  $A = \begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix}$ . (15 pts)

(c) Find the corresponding eigenvectors for each eigenvalue of  $A$ .

2. Given that  $\vec{u} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ , then: (15 pts)

(a) Find the cross product of  $\vec{u}$  and  $\vec{v}$ .

(b) **True or False:**  $\vec{u}$  and  $\vec{v}$  are parallel.

(c) Find the area of the parallelogram generated by  $\vec{u}$  and  $\vec{v}$ .

3. Given the plane  $M: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 7 \\ 1 \end{pmatrix}$  and the line  $l: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} -2 \\ -6 \\ -2 \end{pmatrix}$ :
- (a) Express the plane  $M$  in the form  $ax + by + cz = d$ . (10 pts)

(b) Find the point of intersection of the plane  $M$  and the line  $l$ .

4. Given  $A = \begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & -2 \\ 3 & 2 & 1 \end{pmatrix}$ , find : (15 pts)

(a) The  $\det(A)$ .

(b) Find the cofactor matrix of  $A$ .

(c) Find  $A^{-1}$  using the adjoint of  $A$ .

5. Let  $T : R^3 \rightarrow R^3$  be the reflection about the plane  $x - y = 0$ . Find: (20 pts)

(a) The matrix  $A$  that describes the transformation  $T$ .

(b) The *kernel* of the transformation  $T$ .

(c) All the *fixed points* under  $T$ .

(d) The image of the plane  $2x - y + z = 3$  under  $T$ .

6. (a) Find the  $3 \times 3$  matrix that rotates a vector by  $45^\circ$  around the  $x$ -axis, and then projects it on the  $yz$ -plane. (10 pts)

(b) Use the matrix above, to find the image of the vector  $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ .

7. Consider the triangle with vertices  $A = (1,0,0)$ ,  $B = (3,0,0)$ , and  $C = (2,0,2)$ .

(a) Find the  $4 \times 4$  matrix that first rotates the triangle  $ABC$  by  $90^\circ$  about the  $z$ -axis, and then translates it by  $(0,-5,0)$ .

(b) Use the above matrix to find the coordinates of the shifted triangle.

(c) Make all relevant graphs that show the original triangle, the rotation, and the shifted triangle. (15 pts)

**Some useful formulas:**

(1) **Reflection** about the plane  $ax + by + cz = 0$ :

$$A = \frac{1}{a^2 + b^2 + c^2} \begin{pmatrix} -a^2 + b^2 + c^2 & -2ab & -2ac \\ -2ab & a^2 - b^2 + c^2 & -2bc \\ -2ac & -2bc & a^2 + b^2 - c^2 \end{pmatrix}$$

(2) **Projection** onto the plane  $ax + by + cz = 0$ :

$$A = \frac{1}{a^2 + b^2 + c^2} \begin{pmatrix} b^2 + c^2 & -ab & -ac \\ -ab & a^2 + c^2 & -bc \\ -ac & -bc & a^2 + b^2 \end{pmatrix}$$

(3) **Rotation** about the unit vector  $\vec{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$  by an angle  $\theta$  :

$$A = \begin{pmatrix} a^2(1 - \cos \theta) + \cos \theta & ab(1 - \cos \theta) - c \sin \theta & ac(1 - \cos \theta) + b \sin \theta \\ ab(1 - \cos \theta) + c \sin \theta & b^2(1 - \cos \theta) + \cos \theta & bc(1 - \cos \theta) - a \sin \theta \\ ac(1 - \cos \theta) - b \sin \theta & bc(1 - \cos \theta) + a \sin \theta & c^2(1 - \cos \theta) + \cos \theta \end{pmatrix}$$